Technical Notes

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A New Method for Calculating Ducted Flows

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A NEW one-dimensional method for calculating steady flows in ducts, such as expansion nozzles, shock diffusers, or ejectors, is presented. In this method, the derivation of the flow equations is based on an explicit presentation of the force/momentum balance in the duct. This is the familiar approach for calculating the flow in ejectors. However, the approach requires that the wall pressure forces in the case of a tapered duct, such as an expansion nozzle, are properly accounted for. Ejector calculations mostly circumvent this requirement by assuming either constant pressure or constant area mixing for the ejector.

The momentum law as a universal basis for deriving the flow equations allows a ready account for any effects that affect the force balance in the duct, such as wall friction or duct taper. For the sake of brevity, this Note restricts its derivations to single-flow applications and concentrates in particular on the case of the expansion nozzle, where it compares calculation results with those obtained with the method of Ref. 1, which uses the concept of influence coefficients to calculate flows in ducts. Certain advantages of the present method become apparent. The application of the present method to ejectors has been extensively covered in Refs. 2 and 3.

With A, p, and I denoting the cross-sectional area, static pressure, and flow momentum, respectively, indices 1 and 2 referring to the inlet and exit of the duct, F_f indicating wall friction forces, and assuming uniform pressure in any cross section, the force/momentum balance for a typical duct arrangement as shown in Fig. 1 (supersonic expansion) can be expressed by

$$I_1 = I_2 - A_2(p_1 - p_2) + \int_1^2 (p_1 - p)\,\mathrm{d}A + F_f \tag{1}$$

Introducing mean pressure values, we can make the substitution

$$\int_1^2 (p_1 - p)\,\mathrm{d}A \equiv i(A_2 - A_1)(p_1 - p_2)/2 \tag{2}$$

The factor i in this substitution plays an essential role in the present method to characterize the pressure distribution along a duct. From its definition it follows that it is unity if the pressure changes linearly with the cross-sectional area of the duct. With $i=2$, Eq. (1) represents the case of a supersonic expansion in a sudden duct enlargement and, with $i=0$, the case of subsonic flow in a sudden duct enlargement (shock diffuser). Other implications of this factor will be described later. First, Eq. (1) will be further developed. Introducing the notation

$$A_2/A_1 \equiv t \tag{3}$$

Eq. (1) can be written

$$I_1 = I_2 + (p_2 - p_1)A_2\tau + F_f \tag{4}$$

where

$$\tau = (i/2t) + 1 - (i/2) \tag{5}$$

So far the derivations are not restricted to one-dimensional flow conditions. As long as the previously made assumption about the uniformity of the pressure distribution across the flow is true, Eq. (4) is a consistent presentation of the real flow. In the following, the momenta and the friction force are expressed in terms of Mach numbers. By interpreting the resulting equation as one-dimensional, a relation can be established between inlet and exit momentum. With this approach the subsequent relations are no longer fully consistent with the real flow and the calculated exit momentum can be only an approximation. However, the approach allows a fairly realistic account of the wall friction influence, as seen below.

Using the pipe friction concept, the friction force can be expressed with the help of the pipe friction coefficient c_f, in the form

$$F_f = (c_f L/2D)\rho_2 v_2^2 A_2 \tag{6}$$

with ρ the density, v the velocity, and L/D a length-to-diameter ratio of the duct. For an exact treatment of Eq. (6) the common pipe friction coefficient can be converted to a surface friction coefficient that allows one to calculate the friction force for the tapered duct. This force can be again converted into the form of Eq. (6) with a free choice of reference diameter and a "tapered pipe" friction coefficient. For the segmental approach, described below, where the duct length becomes only a fraction of the duct diameter, the two types of pipe friction coefficients become practically equal and the choice of reference diameter becomes immaterial. For the convenience of the present calculations, D_2 was chosen as reference diameter.

Introducing the Mach numbers for expressing inlet and exit flow momenta by using the well-known relation $I=$

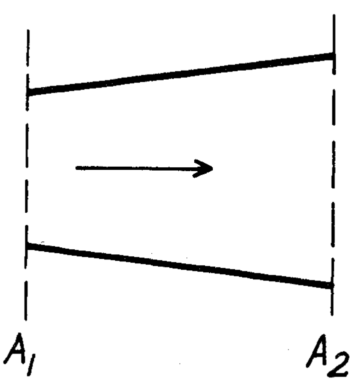

Fig. 1 Flow duct scheme.

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$A\cdot p\cdot\gamma\cdot M^2$, ($\gamma$ is the ratio of specific heats), Eq. (4) can be transformed into

$$\frac{\gamma M_1^2}{t}+\tau=\frac{p_2}{p_1}\left[\gamma M_2^2\left(\frac{c_fL}{2D}+1\right)+\tau\right] \tag{7}$$

Under the one-dimensional interpretation, Eq. (7) contains two unknowns: the exit Mach number M_2 and the pressure ratio across the duct p_2/p_1. Conservation of mass and energy in the duct, as expressed by the relations

$$v_1A_1\rho_1=v_2A_2\rho_2 \tag{8}$$

$$c_pT_1+(v_1^2/2)\pm\Delta Q=c_pT_2+(v_2^2/2) \tag{9}$$

(where c_p is the specific heat at constant pressure, T the temperature, and ΔQ the heat amount), allows one to write an additional condition for determining the two unknowns.

The left side of Eq. (9) can be considered as given (the determination of ΔQ may require some approximations). Since the left side determines the total temperature at the duct exit, one can write

$$c_p(T_2)_0=c_pT_2+(v_2^2/2) \tag{10}$$

where $()_0$ is the stagnation condition.

For the convenience of the present derivations, Eq. (10) is transformed into the well-known form

$$(T_2)_0/T_2=1+M_2^2(\gamma-1)/2 \tag{11}$$

Using also the equation of state, one arrives with Eqs. (8-11) at

$$M_2\left(1+\frac{\gamma-1}{2}M_2^2\right)^{1/2}=\frac{M_1}{t}\frac{p_1}{p_2}\sqrt{\frac{(T_1)_0}{T_1}\frac{(T_2)_0}{(T_1)_0}} \tag{12}$$

For adiabtic flow conditions, the second temperature ratio under the root becomes one. Solving Eq. (7) for p_1/p_2 and introducing it in Eq. (12) leads to the basic equation of the present method

$$M_2\sqrt{1+\frac{\gamma-1}{2}M_2^2}\Big/\left[\gamma M_2^2\left(\frac{c_fL}{2D}+1\right)+\tau\right]=M_1\sqrt{\frac{(T_1)_0}{T_1}\frac{(T_2)_0}{(T_1)_0}}\Big/(\gamma M_1^2+t\tau)\equiv E \tag{13}$$

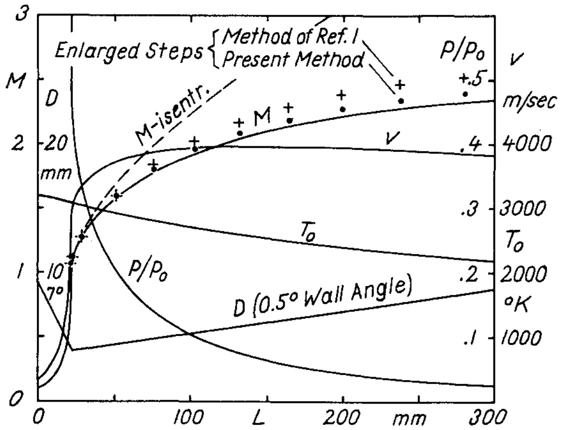

Fig. 2 Mach number M, gas velocity v, pressure ration p/p_0 and total temperature T_0 for expansion of helium in a water-cooled axisymmetric nozzle (diameter D and length L). Plenum conditions: $p_0=100$ kg/cm^2, $T_0=3200$ K; pipe friction coefficient 0.02 for subsonic and 0.015 for supersonic nozzle portion.

Equation (13) can be solved for the exit Mach number

$$M_2^2=\left\{1-\left[\pm\sqrt{1-\frac{\alpha\tau^2}{c}\left(2\frac{c}{\tau}+\frac{1}{\gamma}-1\right)}+\alpha\tau\right]\right\}\Big/(c\alpha\gamma-\gamma+1) \tag{14}$$

with the abbreviations

$$c=1+(c_fL/2D) \tag{15}$$

$$\alpha=2\gamma E^2c \tag{16}$$

The positive and negative roots in Eq. (14) give the subsonic and supersonic solutions, respectively.

The pressure ratio across the duct becomes, with Eq. (7)

$$\frac{p_2}{p_1}=\frac{(\gamma M_1^2/t)+\tau}{\gamma M_2^2c+\tau} \tag{17}$$

For the case of an ejector, the right side of Eq. (13) must be modified to include the momentum of a second fluid and to account for the thermodynamic mixing of both fluids as described in Ref. 2.

The practical use of Eqs. (14) and (17) depends on the knowledge of τ. For an isentropic flow process, the exact value of τ can be found from Eq. (4) by determining the pressure terms in this equation from common isentropic relations. Since wall friction can be assumed, within certain limits, to have a negligible effect on the *shape* of the pressure distribution in the duct, i.e., no essential effect on τ, Eqs. (14) and (17) can be used to calculate flows with friction with τ found from the isentropic case. The present method has been applied with success to ejector pumps by assuming i to be a constant of 0.8 (see Ref. 3). Constant-area supersonic diffusion with friction can be calculated with $\tau=1$. Other applications concern cases where the pressure distribuiton can be assumed to change linearly with the duct cross-sectional area, i.e., where $i=1$. In this case, Eq. (5) gives

$$\tau=(t+1)/2t \tag{18}$$

Thus, τ becomes simply a function of the duct area ratio. The linear distribution, for instance, applies with good approximation to supersonic shock diffusion in slightly tapered ducts.

If one divides an expansion nozzle into sufficiently small segments, the pressure distribution for the individual segment can be made to approach that of a straight line and the flow in the segment can be calculated with τ found from Eq. (18). By proceeding stepwise from segment to segment, the flow in the whole nozzle can be calculated. The practicality of this approach can be readily shown for the isentropic case, which allows an immediate comparison with known values. For a supersonic expansion nozzle (4 mm throat diameter and 7 and 5 deg wall angles for the subsonic, supersonic portions, respectively) with a supersonic area expansion ratio of 19.02, the present method [Eq. (14)] gives for $\gamma=1.4$ an expansion Mach number of 4.669 against 4.665 for the exact theoretical value. The expansion pressure ratio [Eq. (17)] in both cases is 0.00282. The segment length in each calculation step was one-tenth of the local nozzle diameter. The nearly perfect agreement shows the reliability of the approach. The segmental application can be used to vary input data such as the wall friction and heat transfer along the duct.

For a practical example, the present method using τ from Eq. (18) has been applied to the expansion of helium in a very slender nozzle, as actually used to accelerate small particles to very high velocities (2000 m/s). The helium is arc heated and the nozzle is water cooled, incurring unusally high heat losses of about 10 kW/cm^2 in the throat area of the nozzle. The heat-transfer conditions are derived from the Reynolds analogy between the heat transfer and the wall friction.

Figure 2 shows the results of the calculations with flow values, pressure ratio, and total temperature plotted over the nozzle length. The segment size had been reduced in these calculations to the point where its influence on the results practically disappeared. The supersonic Mach number curve demonstrates the influence of the segment size, comparing it also with the condition encountered with the method of Ref. 1. If the segment size for both methods is reduced to the point of no influence, complete agreement exists between both methods. However, the segment size required by the method of Ref. 1 is about five times smaller than that required for the present method; i.e., for the same accuracy, the method of Ref. 1 requires many more steps in the calculation than the present method. By increasing the segment size for the present method by a factor of five, the solid points above the Mach number curve in Fig. 2 result. By using the same segment size for the method of Ref. 1, the Mach numbers marked by the crosses above the solid points are obtained; i.e., for the same enlarged segment size the method of Ref. 1 deviates much more from accurate values than the present method.

The present method has another advantage. It does not become undetermined at Mach 1, as evident from Eq. (13); i.e., no special precautions are necesary for supersonic calculations near the sonic point [for subsonic calculations, inadmissable segment area ratios recognized by imaginary results in Eq. (14) must be avoided].

The flow process least accessible to the present calculation method is subsonic diffusion. In this process, unpredictable flow separation is likely to occur, which, in general, makes a fair estimate of τ very difficult.

References

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Wall Mass Transfer and Pressure Gradient Effects on Turbulent Skin Friction

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Introduction

MOTIVATED by current research on viscous drag reduction, the effects of mass injection and pressure gradients on the drag of surfaces were examined theoretically using boundary-layer and Navier-Stokes codes. Two turbulent flow cases were considered: the effect of spatially nonuniform surface injection with fixed total mass flow rate on a flat surface, and the effect of sinusoidal mass injection and suction on a sinusoidal surface in the presence of a streamwise pressure gradient. For a flat surface the skin friction reduction due to continuous injection is well known; the present study focuses on the effect of spatially varying the injection on flat-plate drag. Little is known regarding the effects of suction and injection on wavy wall surfaces; the present study is also focused on this problem.

Calculations were made for 1.2 m long surfaces, one flat and the other sinusoidal with a wavelength of 30.5 cm. In each case the undisturbed velocity was 15.24 m/s and the initial boundary layer was turbulent at a momentum thickness Reynolds number of 4500. For the flat surface, a finite-difference boundary-layer code[1] was used to determine if spatial variations in mass transfer for a fixed toal flow rate could result in enhanced drag reduction. For the sinusoidal surface, calculations were made with a Navier-Stokes code[2] for a sinusoidal distribution of suction and injection perpendicular to the wall. In both programs mixing length constants of $k=0.41$ and $(\ell/\delta)_{\max}=0.085$ were used in zero-order turbulence closure schemes, and the wall damping factor A^+ was assumed to be the following function of V^+ for low-speed flow[3]:

$$A^+ = 26\exp(-5.9V^+) \tag{1}$$

where $V^+ = v_w/u_\tau$, v = velocity component perpendicular to the wall, and $u_\tau = (\tau_w/\rho)^{1/2}$.

The Navier-Stokes program was checked for coding errors by comparing flat-surface calculations with boundary-layer program calculations for the same input conditions. For no mass transfer the two methods gave essentially the same integrated drag over the 1.2 m surface (see Table 1). Results obtained for an impermeable wall, a flow with constant blowing, and the solution for one cycle of suction and blowing ($v_w/u_\infty = 0.005$ at maximum suction and injection) are shown in Fig. 1. The differences in c_f level in the suction portion of the wave can be attributed to the different methods of solution and are considered reasonable by the authors.

Flat Surface Results

Calculations were made using the boundary-layer code to determine the effect of various spatial blowing variations on flat-plate skin friction reduction. Injection was computed for one, two, and four strips, while adjusting the injection rates to keep the total injected mass constant. A uniform blowing rate of $v_w/u_\infty = 0.001$ over the complete surface defined the total mass injected. That is, values of v_w/u_∞ equal to 0.002, 0.004, and 0.008 were injected over one-half, one-quarter, and one-eighth surface lengths for single-, double-, and quadruple-step injections, respectively. The drag on the 1.3 m (4 ft) long surface element was obtained by integrating the local skin friction coefficient. (No "correction" or accounting was included for the losses associated with collecting and ducting the air. These losses are a function of the injection air source, laminar flow control (LFC) suction being a

Table 1 Effect of injection method on flat-surface drag reduction

Injection method	v_w/u_∞	$C_D \times 10^3$	Drag reduction % of no-injection drag
None	0	2.86	0
		(Boundary-layer code)	
	0	2.81	—
		(Navier-Stokes code)	
Continuous	0.001	2.26	−21.0
Single step	0.002	2.23	−22.1
	0.004	2.27	−20.6
	0.008	2.32	−18.9
Two step	0.002	2.26	−21.1
	0.004	2.30	−19.6
	0.008	2.35	−18.0
Four step	0.002	2.26	−21.1
	0.004	2.31	−19.2
	0.008	2.35	−18.0

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